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EXPLORING THE LOGICAL SPACE IN THE PATTERNS OF CLASSICAL CHINESE MATHEMATICAL ART

Introduction

The Nine Chapters on the Mathematical Art (*Jiuzhang suanshu*) is a very important classical mathematical text in the history of Chinese civilization. The basic text consists of nine chapters, and was probably written no later than 100 B.C.E. The commentary by Liu Hui was done in the third century C.E. and that by Li Chunfeng was done in the seventh century C.E. The role of this book, as the translators of the *Nine Chapters*, Shen Kangshen, John Crossley, and Anthony Lun point out, is "somewhat similar to Euclid's *Elements of Geometry* in the West."[1] *The Nine Chapters on the Mathematical Art* shaped the framework of Chinese mathematical art the way Euclid's *Elements of Geometry* did in the West.

However, in this article, I would like to call attention to the differently structured logical space in Chinese mathematical art, which was clearly represented in the patterns of the *Nine Chapters*. This kind of structure of logical space is very different from that in the Euclidean tradition. My standpoint is that Chinese mathematicians' reasoning was based on a very different presumption. The presumption in the Euclidean tradition is that there is a fixed order in this world, and the goal of doing mathematics is to represent the beauty of this rational order. The presumption of the Chinese mathematicians is that there is no fixed order in this world. For them, things are changing all the time. Following this presumption, any universal rule, which aims to represent the fixed order in the world, is not important, or for that matter, even impossible. The mathematical art in Chinese culture is akin to conversational reasoning. The logic that Chinese mathematicians followed in this kind of conversational reasoning deals with the relations among particulars in present practice. The aim of this kind of reasoning is to represent the harmony of relations among particulars at the moment. The achievement

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of harmony requires subjective participation guided by a common aesthetic sense. Therefore, the goal of the *Nine Chapters* is not merely to solve problems but to represent the aesthetic order among particulars. The beauty of Chinese mathematical art is, in a way, similar to the beauty of poems; it requires the participation of subjects.

There are three sections in this article. In the first section, I shall use the patterns established by ancient Chinese mathematicians in the *Nine Chapters* for finding algorithms as my point of departure, and explore the logical space in these patterns. I shall discuss the characteristics of the logical space shown in these patterns. The role of the time, *now*, in these patterns is emphasized. The logical space in Chinese mathematical art is structured in the present time only. In the second section, I shall discuss how this logical space is manifested in particular problems and particular ways to solve them in present practice. I shall point out that the beauty of Chinese mathematical art is obscured when Shen, Crossley, and Lun inconsistently translate the two key terms in these patterns, *jinyou* (now, there is . . .) and *shu* (art/method), for the purpose of making them conform to Western mathematical terminology. In the last section, I shall address the beauty of Chinese logic in Chinese mathematical art. I shall demonstrate that, in fact, these patterns showed a unique logical structure, which is based on the principle of kind (*lei*). The logical reasoning of kind (*lei*) can be described as a net, which represents the main characteristic of Chinese logic.

My conclusion is that the whole text of the *Nine Chapters* was constructed strictly according to Chinese logic, which forms a very different logical space from that in the Euclidean tradition. The fruitful contribution of applications in the *Nine Chapters*, which is not considered to be so important in the Euclidean tradition, is related to Chinese logic—a thinking structure of a "net." Understanding the structure of Chinese logical space is the main avenue to understanding the work of Chinese mathematicians.

Logical Space and the Role of Time (*Now*) in Patterns of the *Nine Chapters*

In the preface to the *Nine Chapters on the Mathematical Art*, immediately after claiming that the role of Chinese mathematical is similar to that of Euclid's *Elements of Geometry* in the West, the translators, Shen, Crossley, and Lun point out:

> The *Nine Chapters* has always been more involved in methods for finding an algorithm to solve a problem, so that its influence has been both pedagogical and practical. Instead of theorems, which Western readers

> are accustomed to find in works written in the Euclidean tradition, the *Nine Chapters* provides algorithmic rules . . . Liu's proofs are not written in an axiomatic style.[2]

Shen, Crossley, and Lun left the above philosophical question unsolved, although their translation did open the way to a fruitful problem worthy of contemplation in the West. To investigate the deep reason why ancient Chinese mathematicians were more involved in the methods for finding an algorithm to solve practical problems one by one, than in developing theorems, one needs to look closely at the logical space represented by the patterns in the *Nine Chapters*.[3] From there, one can further reveal the underlying presumption of Chinese logic. In fact, only when one understands that the logical spaces in Chinese logic are differently structured from that of Euclidean tradition, can one finally understand why ancient Chinese mathematicians were able to solve many difficult mathematical problems while neglecting to demonstrate the steps. Only when one sees that a whole different set of logical rules, based on the presumption of a changing world, functioned in Chinese mathematical arts, will one be able to appreciate the original beauty of Chinese mathematical art.

To uncover the logical structure and presumption in Chinese mathematical art, I would like, first of all, to call attention to a few important and interesting features of the *Nine Chapters*:

1. None of the mathematical terms in the *Nine Chapters* have a given definition.
2. No demonstrations between a given problem and an answer are offered.
3. The 246 problems in the *Nine Chapters* mostly begin with the phrase *Jinyou*, which means "Now, there is . . ." *Jinyou* is a general way to form patterns in the *Nine Chapters*.

Second, I would like to briefly summarize the patterns according to which the mathematical problems in the *Nine Chapters* are organized:

Pattern 1:
Now, there is (*jinyou*) . . . Tell (*qiu*):
Answer (*da*):
Art/Method (*shuyue*):

For example, see problem 12 in the chapter "Excess and Deficit":

> **Now there is** a wall 5 *chi* thick, two rats tunnel from opposite sides. On the first day the big rat tunnels 1 *chi*; the small rat also tunnels 1 *chi*. The big rat doubles its rate daily, the small rat halves its rate daily. **Tell:** the number of days till the two rats meet. The distances tunneled by each.
> **Answer:** 2 2/17 days; the big rat tunnels 3 *chi* 4 12/17 *cun*; the small rat tunnels 1 *chi* 5 5/17 *cun*.

Method [art]: Assume 2 days, deficit 5 *cun*; assume 3 days, excess 3 *chi* 71/2.[4]

Pattern 2:

The name of an art/method (*shu*) or a rule (*fa*)
Art/Method (*shuyue*):
Now, there is (*jinyou*) . . . Tell (qiu):
Answer (*da*):
Art/Method (*shuyue*):

For example, see problem 1 in the chapter "Millet and Rice":

***Jinyou* Rule:**[5]
Art/Method: Taking the given number (*souyoushu*) to multiply the sought rate (*souqiulu*). [The product] is the dividend. The given rate (*suoyoulu*) is the divisor. Divide.
Now, there is millet 1 *dou*, which is required as hulled millet. **Tell:** how much is obtained?
Answer: As hulled millet: 6 *sheng*.
Art/Method: Taking millet is required as hulled millet, multiply by 3, divide by 5.[6]

Pattern 3:[7]

Now, there is (*jinyou*) . . . Tell (qiu):
Answer:
Art/Method:
Another art/method:
Another art/method:
Another art/method:

For example, see problems 31 and 32 in the chapter "Field Measurement":

Now, there is a circular field, the circumference is 30 *bu* and the diameter 10 *bu*. **Tell:** what is the area?
Answer: 75 *bu*.
There is another circular field, the circumference is 181 *bu* and the diameter 60 1/3 *bu*. **Tell:** what is the area?
Answer: 11 *mu* and 90 1/12 *bu*.
Art/Method: Multiplying half the circumference by the radius yields the area of the circle in [square] *bu*.
Another art/method: One-fourth the product of the circumference and the diameter.
Another art/method: One-fourth the product of three times the diameter squared.
Another art/method: The circumference squared and divided by 12.[8]

The above features and patterns in a mathematics text are very unfamiliar to a westerner. The logical steps between a problem and an answer are missing here. The function of "art/method" (*shu*) seems unclear. It is noteworthy that Shen, Crossley, and Lun's translation and commentary of the *Nine Chapters* did a useful job in filling in the missing logical steps between a problem and an answer, and matching the Chinese art/method with the Western mathematical rules; their work made this text understandable for Western readers. But the original beauty of Chinese logic, which enlivens the Chinese mathematicians' work from the beginning to the end, is not clearly brought out.

Third, I will explain here the kind of logic that guided the traditional Chinese mathematicians' intelligent work, and show the kind of logical space inherent in the Chinese patterns. "Logical space" here, in my understanding, is the space of logical possibilities. These possibilities are marked by the boundaries between concepts. The boundaries between concepts are shaped by underlying logical relations. Different languages may structure logical relations in different ways. Chinese culture structured its logical relations in a unique way. With regard to logical relations, let us look at the *space* of logical possibilities in these patterns. Logic is made problematic in different languages not only by the fact that logical space may be divided up in different ways—for example, the meanings of two terms in two different languages cannot perfectly match—but also by the fact that this space may itself be *structured* differently. For instance, the possibilities of structuring logical relations might be very different if situated in a coordinate system where variables are differently defined, or if the language-game players hold a completely different understanding of time, space, orders, movement, and other fundamental concepts.

When James Griffin tries to comment on Wittgenstein's logical space, he says:

> In a given co-ordinate system putting two numbers together defines a point; in a given language putting two names together makes a statement. In this way, languages are a kind of co-ordinate system. And as there are different co-ordinate systems as a result of choosing different points of origin, different scales, and so forth, there are different representational forms in language.[9]

To make generalizations about the above three patterns in the *Nine Chapters*, we may focus on the first one since the second and third patterns are merely variations of the first. Pattern 1 can be taken as the general pattern in the *Nine Chapters*:

> Now there is [a problem] . . . tell:
> Answer:
> Method/Art:

The first phrase here is "Now, there is . . ." (*jinyou*).[10] If one takes a close look at the above pattern, one can easily see that "time" plays an important role in each mathematical problem-solving procedure. Almost all of the problems in the *Nine Chapters* start with the assumption, "Now, there is . . ." (*jinyou*), which is a good starting point for us to explore the logical space in these patterns.

To the extent that the time, "now" (*jin*), is involved, the problems in which Chinese mathematicians are interested are particular ones, such as those that arise during a face-to-face conversation in the present. In other words, Chinese logical space is structured in the time, "now." Chinese people are only concerned with the logical relations that exist in the present practice, not something beyond the present time, such as "universal truth." It is a fact that unreflective practices in reasoning (*logica utens*) are present in Chinese daily life. Chinese people have pondered this *logica utens*, and attempted to represent those practices and improve on them through theory (*logica docens*). But Chinese theory differed from Western logic; it had a different set of methods and principles. These methods and principles are represented in the work of Chinese mathematicians in their discussions of the practical mathematical problems.

The underlying presumption of Chinese logic, which ancient Chinese mathematicians followed, is that everything in this world is in a continuous changing process. There is no fixed order by which one can organize the position of things in the world. Even mathematics does not aim at searching for the fixed order or absolute truth, but rather at solving problems in practice. In his preface to the *Nine Chapters*, Liu Hui stated very clearly that the tradition of Chinese arithmetic had its base in binary arithmetic, that is, *yin-yang*, the four diagrams, the eight trigrams, and the sixty-four hexagrams.[11] Based on the *yin-yang* principle, anything can be both him/her/itself and something else. Everything has its role and duty in associations only. This is how a changing and harmonious world should be. When time—how things are *now* and will be for some time to come—is involved as an important element, ideally, an understanding of temporal stability in the Chinese sense can be achieved. This is the place where mathematicians can work. In other words, Chinese mathematicians' objective is to solve problems one by one in the present time—*now*—not to find a universal formula that can solve all the problems posed in similar situations regardless of their complexity or the time at which they might arise. The logical relations that are structured in the present time, *now*, involve the employment of an aesthetic order. This aesthetic order is the order that Chinese mathematicians seek.

Logical Space and Solving Particular Problems in Present Practice

The phrase *jinyou* is crucially important to understanding the patterns in the *Nine Chapters.* Having discussed the role of time, the *now* (*jin*), in the patterns, the meanings of *you* in the phrase of "now, there is . . ." (*jinyou*) should be clarified. Shen, Crossley, and Lun translate *jinyou* according to individual cases. Sometimes, they translate it as "now, there is . . . [a problem]"; at other times, they translate it as "now, given . . . [a problem/a rule].[12] I think we need to be careful with the second translation, because the Chinese character *you* does not hold the meaning of "given."

The character *you* in Chinese means that a happenstance exists or shows itself, or that something is possessed. The original character *you* is written in such a way that the top part is a hand and the bottom part is a moon. In the *Shuo Wen*, an early Chinese lexicon, Xu Shen says, "*You* is the thing that does not always exist. *Spring and Autumn* has an explanation: [for example,] the happenstance of a solar eclipse or lunar eclipse."[13] It is clear that *you* in the *Nine Chapters* does not hold the meaning of something that is given by mathematicians theoretically, but that it means a concrete problem that occasionally exists as a special event, in a particular time and space.

If one holds the presumption that there is a fixed order in this world and that things have their stable positions, then the notion of "given a problem" or "given a rule" can make sense in mathematical reasoning. In fact, in the Western worldview, mathematics has been understood as an effective way to represent the beauty of this order. Logical discourse, in which mathematicians are interested, aims at the discovery of Truth, which stands above us all and serves as a standard by which we are constrained. As Bertrand Russell claims:

> Mathematics is, I believe, the chief source of the belief in eternal and exact truth, as well as in a super-sensible intelligible world.[14]
>
> The theory is developed in Euclid, and has great logical beauty. The method is purely deductive, and there is no way, within it, of testing the initial assumptions.[15]

In Western mathematical terminology, the phrase "given [a problem]" can be a starting point for getting into the process of searching for this order. The phrase "given [a rule]" can guide one in finding out the solution of a given problem logically. These phrases are all based on the presumption that there is a fixed order for which we may search.

As I have pointed out, Chinese mathematicians made a very different presumption, which is that there is no fixed order in this world; things are changing all the time. Mathematics aims to represent the harmony of relations among particulars at the moment. The logic that Chinese mathematicians followed deals with the relations among particulars in present practice. Therefore, the role of mathematics in the Chinese worldview is a little more complex than that in the West. As Liu Hui says at the beginning of the preface of the *Nine Chapters*:

> In the olden days, Fu Xi created the eight trigrams of remote antiquity in order to communicate the virtues of the gods and show the parallels of the trends of the myriad things on earth, inventing the nine-nines algorithm to co-ordinate the changes of the hexagrams.[16]

As a conversational reasoning, the achievement of harmony requires our participation guided by a common aesthetic sense. The goal of the *Nine Chapters* is not merely to solve problems but to represent the aesthetic order among particulars. The beauty of Chinese mathematical arts is, to some degree, similar to the beauty of poetry; it requires the participation of subjects. The beauty of harmony cannot be simply "given." In short, "now, given [a problem/a rule]" is a Western mathematical term, not a Chinese one.

The second term, *da*, in the above patterns is a concrete answer to a concrete problem. Sometimes, there can be more than one answer, but there are no demonstrations that show how one can arrive at the answer.

The third term in the patterns, *shu*, is an interesting concept. In Shen, Crossley, and Lun's translation of the *Nine Chapters*, *shu* is rendered as "method/art" (in the chapter "Millet and Rice"), or "rule/general rule" (in the chapter "Field Measurement"). It seems to me that translating *shu* as "rule/general rule" is problematic. According to the *Shuo Wen*,[17] the original meaning of *shu* is "the way/path in a village."[18] In other words, it is a way/path that connects two things. Let's look at the situation in Pattern 2:

> The name of an art/method (*shu*)
> Art/Method (*shuyue*):
> Now, there is (*jinyou*) . . . Tell (*qiu*):
> Answer (*da*):
> Art/Method (*shuyue*):

It starts with the name of an art/method (*shu*), followed by the content of the *shu*. Then, it conforms to the general pattern, as in Pattern 1. Liu Hui calls the name of an art/method, and art/method itself, *dushu*. *Du* means "capital city." Therefore, *dushu* means the way or street in the capital city. It is correct to state that the latter is more important than the former. But, clearly, *shu* is different from a "*rule*," and *dushu* is different from a "general rule," which is the term that Shen, Crossley, and Lun sometimes use to translate *shu* and *dushu*.[19]

Again, let us look at Pattern 3:

> Now, there is (*jinyou*) . . . . Tell (*qiu*):

Answer:
Art/Method:
Another art/method:
Another art/method:
Another art/method:

If we understand *shu* as a "way/path," we can see that Pattern 3 shows that there can be more than one *shu* (way/path) that can link two things—a problem and an answer.

Sometimes Shen, Crossley, and Lun translate *jinyou* as "now, there is [a problem/a rule] . . ." and *shu*, as "art" or "method."[20] At other times, they translate *jinyou* as "now, given [a problem/a rule] . . ." and *shu*, as "rule" or "general rule." It is inappropriate to translate *jinyou* as "now, given . . ." and *shu* as "rule" or "general rule"; it will confuse readers, preventing them from seeing the special patterns and the logical structure of these patterns in the *Nine Chapters.*

To sum up, the above discussion of the three patterns in the *Nine Chapters* shows that since the logical space in these patterns is structured in the time, *now*, the logical relations involved in these patterns are the relations between particulars, that is, a particular problem and a particular answer to the problem. Mathematics is the art of showing the ways or paths that link these particulars; it is a picture of harmony, which Chinese mathematicians have always looked for since the time when Fu Xi is thought to have created the eight trigrams. In a changing world, the best hope for Chinese mathematicians is to clarify the relations among particular things in a net of associations at the present time, *now*.

The Principle of Kind (*lei*): A Chinese Way of Reasoning

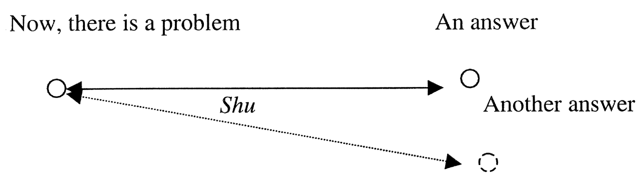
Looking at the *Nine Chapters* and Euclid's *Elements of Geometry* at the level of logic, we see that their fundamental difference may shed light on the divergent ways of logical thinking in the two different cultures, Chinese and Western. In terms of a metaphor, the logical space of Chinese mathematical arts is structured as a net. Unlike the ancient Greeks, who insisted on deductive reasoning, and thus made mathematics abstract, the Chinese made mathematics practical and personal. Unlike Porphyry's trees, which present a rational beauty of unidirectional order, or the Aristotelian classificatory system, a hierarchical structure of names for kinds of things organized into genera and species, which identifies objects by either essence or membership, or both, the Chinese aesthetic order, as represented in the *Nine Chapters,* is bidirectional. Working within this framework, if one asserts that something is unchanging, one will be accused of failing to understand the development of the world. As Shen, Crossley, and Lun point out:

> Euclid's Elements, being based in the ancient Greek tradition of philosophy, is very different indeed in both context and form from traditional Chinese mathematics. It is almost certain that to the Chinese it seemed very alien in its approach to mathematics.[21]

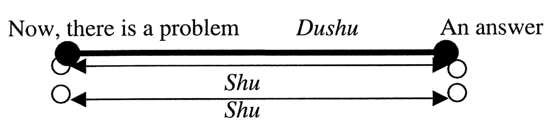
Chinese mathematical art is not based on deductive reasoning. At the boundary of logic and aesthetics, an adjustable standard is required. This adjustable standard is "kind" (*lei*). Chinese mathematical art is based on a special kind of reasoning—the relations of kind (*lei*). Kind (*lei*) deals with the bidirectional aesthetic order effectively. Mathematical thinking focuses on revealing the relations among kinds. If similar kinds of relations were established, then reasoning about one of the kinds would be valid for the other. For example, a problem about how many *mu* of a triangle-shaped field and an answer of the total *mu* of this field can be considered as a similar kind (*lei*) of things, and method/art (*shu*) establishes the relation or path between the two things. Deductive steps are not important for Chinese mathematicians; the important thing is to find harmonious relationships in a bidirectional order.

Using charts as those below, we may show the logical space in the above three patterns, with the following structures:

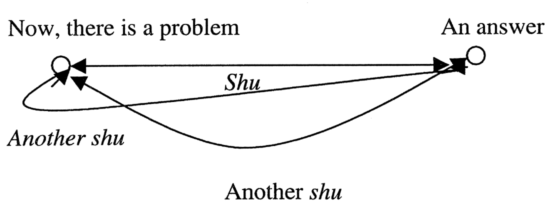
Pattern 1:



Pattern 2:



Pattern 3:



The above charts show that whether it is *shu* or *dushu*, its role is always to deal with the relationship in a bidirectional order. Certain kinds of relationships are criteria for categorizing problems, answers, and method/art. The *Nine Chapters*, in fact, are nine kinds of relations, which are categorized according to the principle of kind (*lei*), such as "Field Measurement," "Millet and Rice," "Short Width," "Construction Consultations," and so on. In the preface of the *Nine Chapters*, Liu Hui says:

> [I] observed the division between the dual natures of yin and yang which sum up the fundamentals of mathematics. Through investigation, I suddenly intuit its true meaning therein, which allows me to collect my ideas and take the liberty of commenting on it. *Things are related as kinds; each of them can be traced back to its own kind*.[22] [emphasis mine]

A good mathematician is one who is good at expanding categories or kinds (*tong lei*). In Mary Tiles and Jinmei Yuan's essay, "Opposition and the Geometry of Logical Space," there is a discussion of the principle of kind (*lei*) in Chinese mathematical art:

> In the *zhou bi suan jing*, an earlier astronomical text, there is a discussion between Rong Fang, in the role of pupil trying to understand how to use mathematics to do astronomy, and Chen Zi, in the role of teacher. When Rong Fang confesses that he isn't getting anywhere Chen Zi tells him: "You thought about it, but not to [the point of maturity]. This means you have not been able to grasp the method of surveying distances and rising to the heights, and so in mathematics you are unable to extend categories [*lei*]" (Cullen, p. 71).
>
> Here the expression for extending categories is *tong lei*; it refers to the ability to see the general principle underlying many apparently different examples. Chen Zi goes on to say: "What makes it difficult to understand the methods of the Way is that when one has studied them, one [has to] worry about lack of breadth. Having attained breadth, one [has to] worry about lack of practice. Having attained practice, one [has to] worry about lack of ability to understand. Therefore one studies similar methods in comparison with each other, and one examines similar affairs in comparison with each other. This is what makes the difference between stupid and intelligent scholars, between the worthy and the unworthy. Therefore, *it is the ability to distinguish categories in order to unite categories* (*lei yi he lie*) which is the substance of how the worthy one's scholarly patrimony is pure, and how he applies himself to the practice of understanding" (Cullen, pp. 71–72; emphasis ours).[23]

Therefore, categorizing according to kind (*lei*) constructs a Chinese framework of logical relationship. This characteristic of Chinese logic explains why Chinese mathematicians are not interested in establishing deductive steps but simply in pointing out the *shu*—the way/path, which tells us the relations between a problem and an answer, and lists a set of similar *shu* under a *dushu.*

Conclusion

Chinese mathematical art aims to clarify practical problems by examining their relations; it puts problems and answers in a system of mutual relation—a *yin-yang* structure for all the things in a changing world. The mutual relations are determined by the *lei* (kind), which represents a group of associations, and the *lei* (kind) is determined by certain kinds of mutual relations. The logical space in the patterns of the *Nine Chapters* shows a bidirectional aesthetic order in the time, *now*. To describe the model of logical relations in this kind of bidirectional order of a group of associations, I borrow Umberto Eco's term, "net," in terms of a metaphor. In Eco's words: "The main feature of a net is that every point can be connected with every other point, and where the connections are not yet designed, they are, however, conceivable and designable. A net is an unlimited territory . . . the abstract model of a net has neither a center nor an outside."[24] The structure of Chinese logical thinking is like a net. The model of a net will offer us guidance in understanding the logic of Chinese mathematical art.

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Endnotes

1. Kangshen Shen, John N. Crossley, and Anthony W. C. Lun, *The Nine Chapters on the Mathematical Art: Companion and Commentary* (Beijing: Oxford University Press and Science Press, 1999), p. 1.
2. Ibid., p. vii.
3. "Logical space" is one of Wittgenstein's metaphors. The essence of the metaphor is similar to the situation in mathematics, namely, a two dimensional coordinate system putting two numbers together, which determines a point. In a given language system, putting terms together makes a statement. In this way, languages are like a kind of logical coordinate system. Different coordinate systems are a result of choosing different scales and different points of origin, which produces different representational forms in the world. A picture has logico-pictorial form in common with that which it depicts. A picture depicts reality by representing a possibility of existence and nonexistence of states of affairs. A picture represents a possible situation in logical space" (Wittgenstein: *Tractatus Logico-Philosophicus* [Atlantic Highlands, NJ: Humanity Press International, Inc., 1988] 2.19–2.202, p. 10).
4. Shen, Crossley, and Lun, *Nine Chapters on Mathematical Art*, p. 369.
5. Shen says, "All of the first 31 problems in this Chapter begin with *Jinyou*, hence the name of the rule. The actual meaning of the *Jinyou* Rule is the Rule of Three" (ibid., p. 142).

   Sometimes, Shen also translates "Art/Method," *shu* as "The General Rule/Rule for . . ." As he says, he does this to making the book understandable to Western readers; he tries to match Western mathematical concepts. But in the original Chinese version of the *Nine Chapters*, the word used is always *shu*. No difference is pointed out between the so-called General Rule, or Rule or method. Shen translates the word *shu* differently according to the individual cases. On the one hand, it does make the *Nine Chapters* read easier for westerners in some sense, but on the other hand, it keeps readers from seeing the pattern of the *Nine Chapters.*

6. Jimin Li, *An Examination and Correction on the Nine Chapters* (Sanxi: Science and Technology Press, 1993), pp. 176, 177.
7. Sometimes, after a common pattern 1, one or more other art/method (*shu*) for solving the problems is pointed out, but neither a universal formula nor a deductive proof is given. (Lui's commentary sometimes adds explanations if he thought them necessary.)
8. Li, *Examination and Correction*, pp. 149–154.
9. James Griffin, *Wittgenstein's Logical Atomism* (Oxford: Oxford University Press, 1964), p. 103.
10. In some chapters, Shen translates "*Jinyou*, Now, there is . . . " as "Now, given . . ., " but I do not think that the Chinese word *you* can be translated as "given." "Now, given" is a translation that aims to match the Western mathematical terms, but it loses some important elements in the original Chinese. I shall discuss the difference between the two translations in the second section of this article.
11. See Shen's translation in Shen, Crossley, and Lun, *Nine Chapters on Mathematical Art*, pp. 52–53.
12. See, for example, Shen, Crossley, and Lun, *Nine Chapters on Mathematical Art*, pp. 64, 70, 408, 410.
13. Xu Shen, *Shuo Wen*, in *Suowenjiezi* (Beijing: Zhonghua Shuju, 1963), p. 141. Translation is mine.
14. Bertrand Russell, *A History of Western Philosophy* (New York: Simon and Schuster, 1945), p. 210.
15. Ibid., p. 211
16. Jimin Li, *Examination and Correction*, pp. 176, 177.
17. See Xu Shen, *Shuo wen jie zi*, p. 51.
18. Ibid., p. 44.
19. See, for example, Shen, Crossley, and Lun, *Nine Chapters on Mathematical Art*, pp. 57, 141.
20. See, for example, ibid., p. 57.
21. Ibid., p. 21.
22. Jimin Li, *Examination and Correction*, p. 126.
23. Mary Tiles and Jinmei Yuan, "Opposition and the Geometry of Logical Space," unpublished essay, Department of Philosophy, University of Hawaii at Manoa, 2001; see Christopher Cullen, *Astronomy and Mathematics in Ancient China: The Zhou bi suan jing* (Cambridge, U.K.: Cambridge University Press, 1996).
24. Umberto Eco, *Semiotics and the Philosophy of Language* (London: Macmillan, 1984), p. 81.

Chinese Glossary

Da	答
Dou	斗
Du	都
Dushu	都术
Lei	类
Lei yi he lie	类 以 合 类
Li Chunfeng	李淳风
Li, Jimin	李继闵
Liu Hui	刘徽
Jin	今
Jinyou	今有
Jinyou Shu	今有术
Qiu	求
Sheng.	升
Shu	术
Shuo Wen	说文
Shuyue	术曰
Souyoushu	所有数
Souqiulu	所求率.
Suoyoulu	所有率
Jiuzhang Suanshu	<<九章算术>>
Tong lei	通类
You	有
Xu Shen	许慎 [汉]
Zhou Bi Suan Jing	<<周髀算经>>